

Radio-Frequency Technique for Investigation of Quantum Properties of Superconducting Structures.

E. Il'ichev

Motivation

Demonstration the simplest quantum algorithm by making use of **flux qubits**
Problem of quantum measurements.

Team



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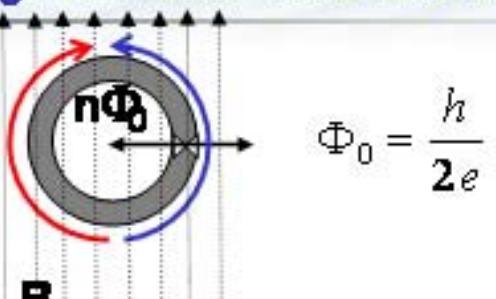
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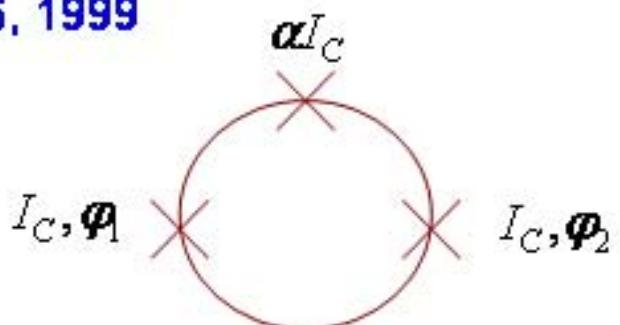
Single-junction interferometer



- Can exhibit MQT (A. Leggett et al.)
- Useful device for memory cells
- However $\beta = 2\pi L I_C / \Phi_0 > 1$

Solution – for $N > 3$ the ring exhibits hysteresis even for negligible loop inductance (T. Yamashita et al., J. Appl. Phys. 19, 2519 (1979).)

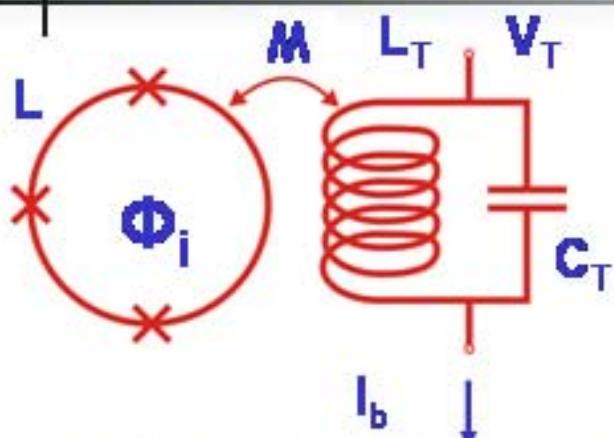
Therefore this structure can be used as qubit J.E. Mooij et al.,
Science 285, 1036, 1999



Outline

- **Readout concept**
- **Two coupled qubits**
- **Basic idea of the Adiabatic Quantum Computation**
- **Three coupled qubits**
- **Ferromagnetic and antiferromagnetic coupling**
- **Four coupled qubits with FM and AF coupling**

Readout concept

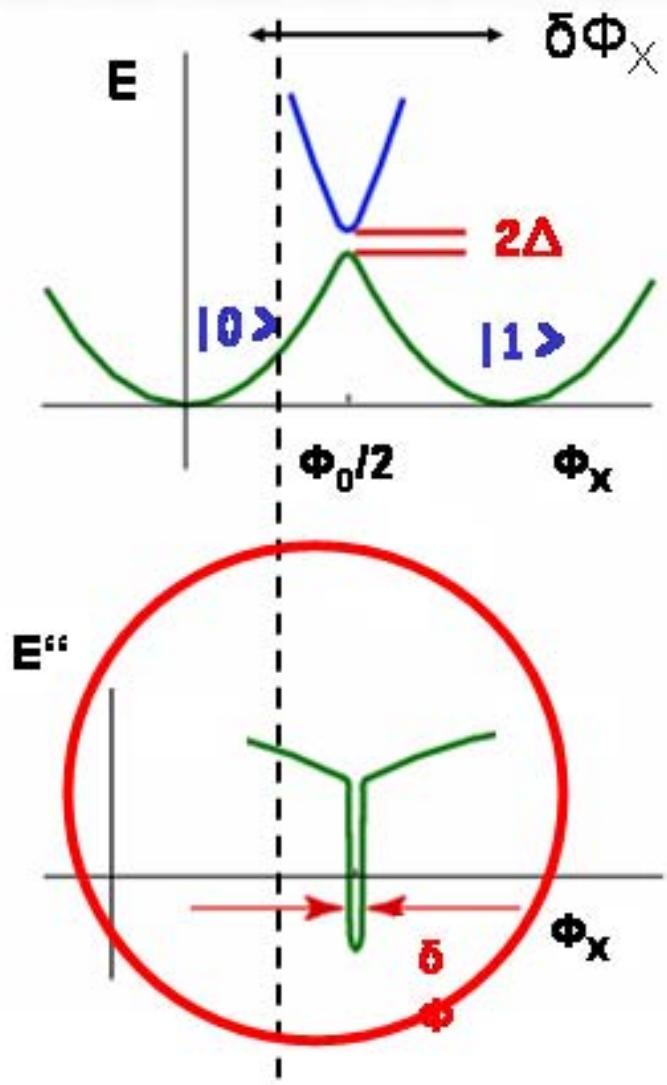


Θ -phase shift between I_b and V_T

$$\tan \theta = k^2 QL \frac{d^2 E}{d\Phi^2}$$

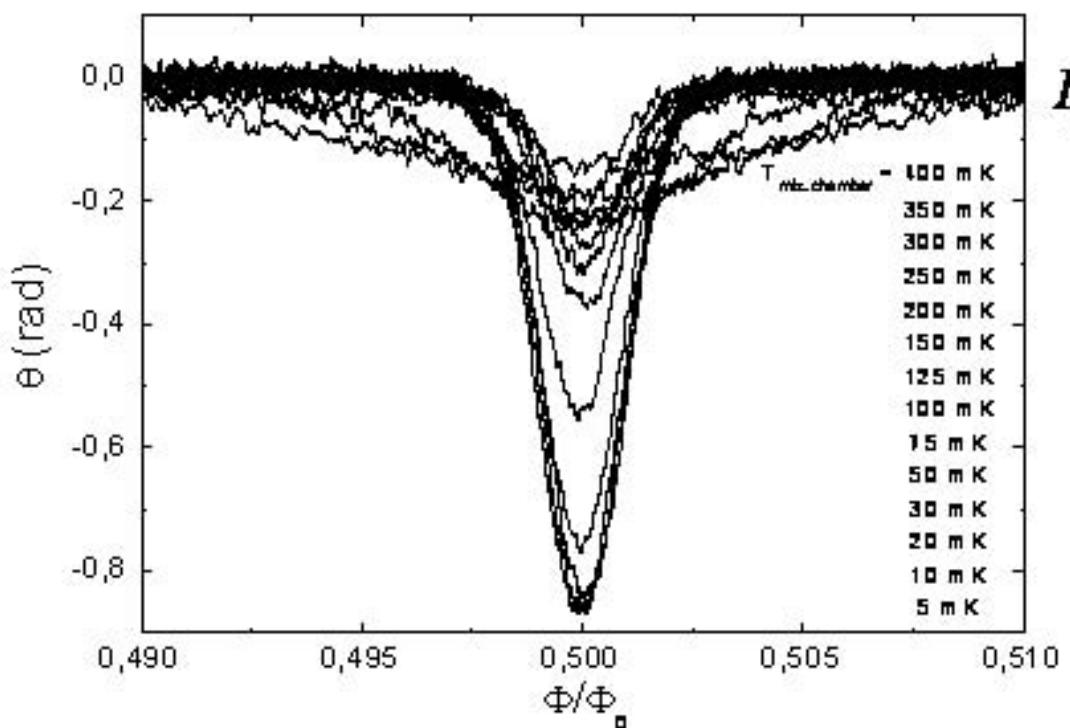
Dip:

evidence for quantum regime
access to qubit parameters
nondemolition readout of qubit



Results

I_b = 77 pA



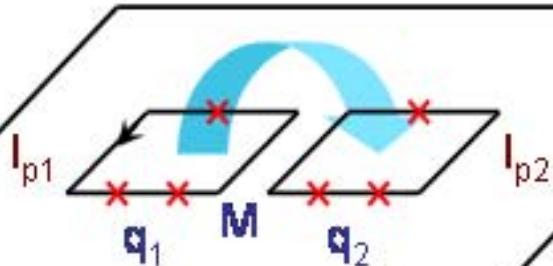
$$I_p = \frac{(\tan \Theta)_{MAX}}{k^2 Q L_q} \cdot \frac{\Phi_0 f_{FWHM}}{2\sqrt{2^{2/3} - 1}}$$

$$\Delta = I_p \cdot \frac{\Phi_0 f_{FWHM}}{2\sqrt{2^{2/3} - 1}}$$

$$I_p = I_c (1 - 1/4 \alpha^2)^{1/2}$$

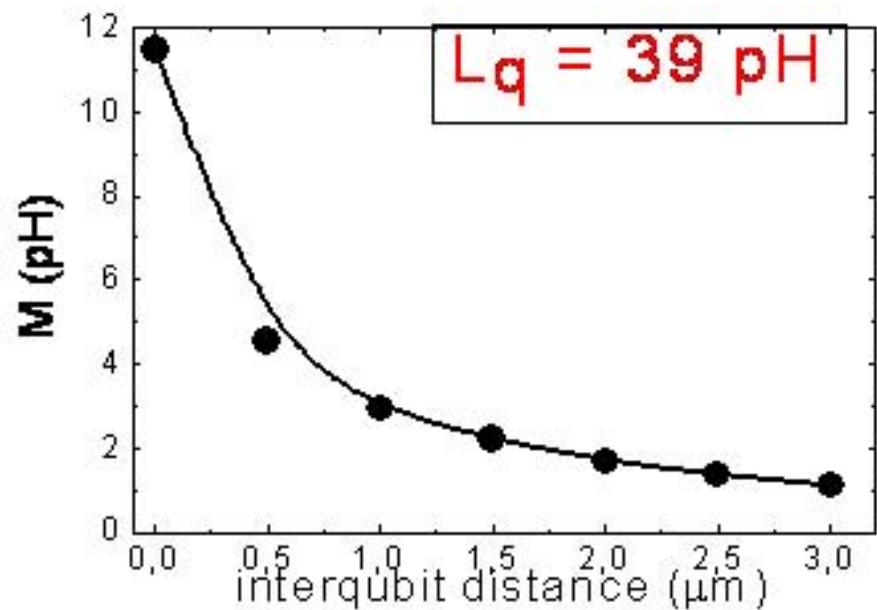
Ya. S. Greenberg et al., PRB 66, 214525 (2002)
M. Grajcar et al., PRB 69, 060501(R) (2004)

3JJ Flux-qubit; Inductive Coupling



Interaction energy: $J = MI_{p1}I_{p2}$

Hamiltonian: $H_{\Sigma} = H(q_1) + H(q_2) + J\sigma_1^z\sigma_2^z$
 $H(q_i) = \varepsilon_i\sigma_i^z + \Delta_i\sigma_i^x$



Drawback

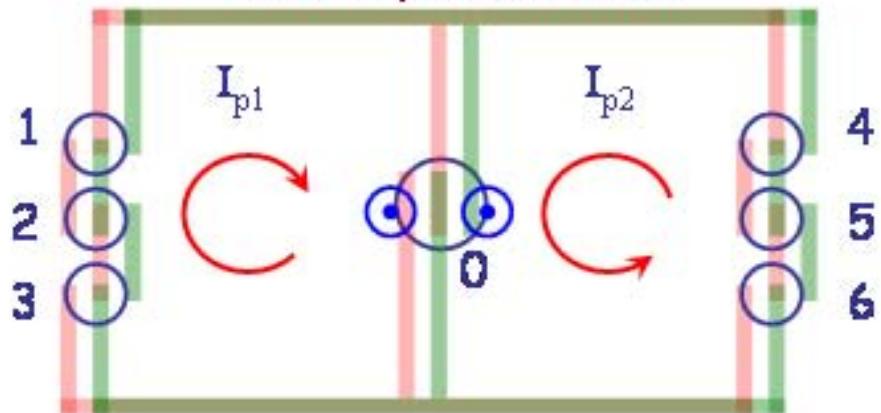
The "size" problem: Coupling energy cannot be made strong enough for small L_q

Interaction energy can be increased if

1. Qubits have shared leg
2. Persistent currents increase

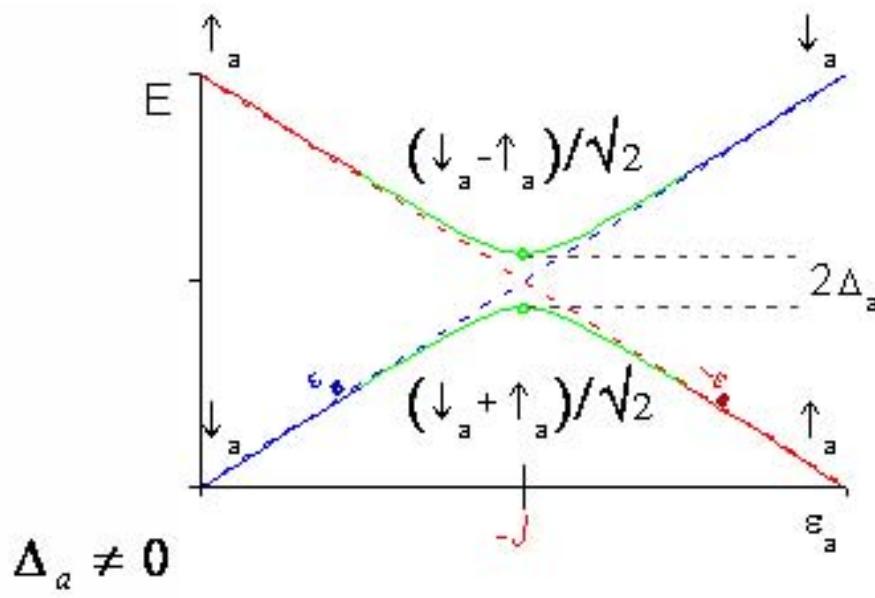
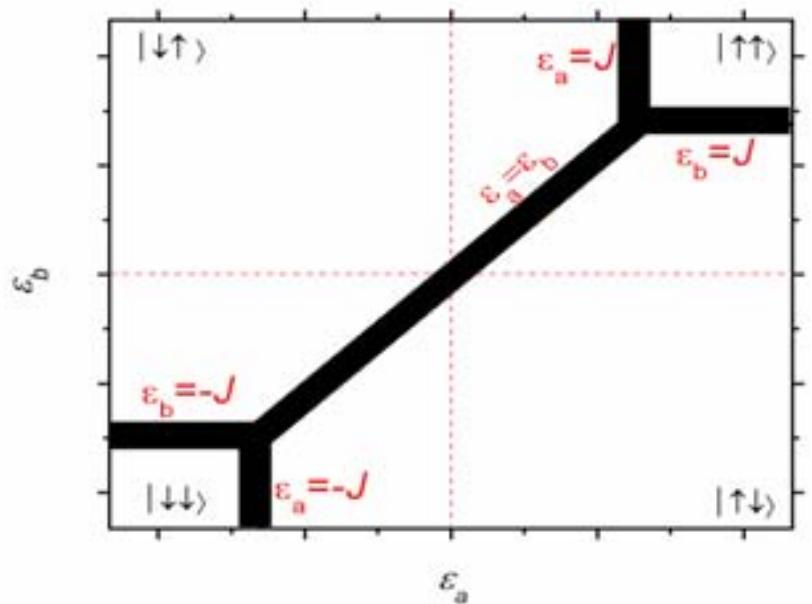
Coupling through a shared Josephson junction

Our implementation

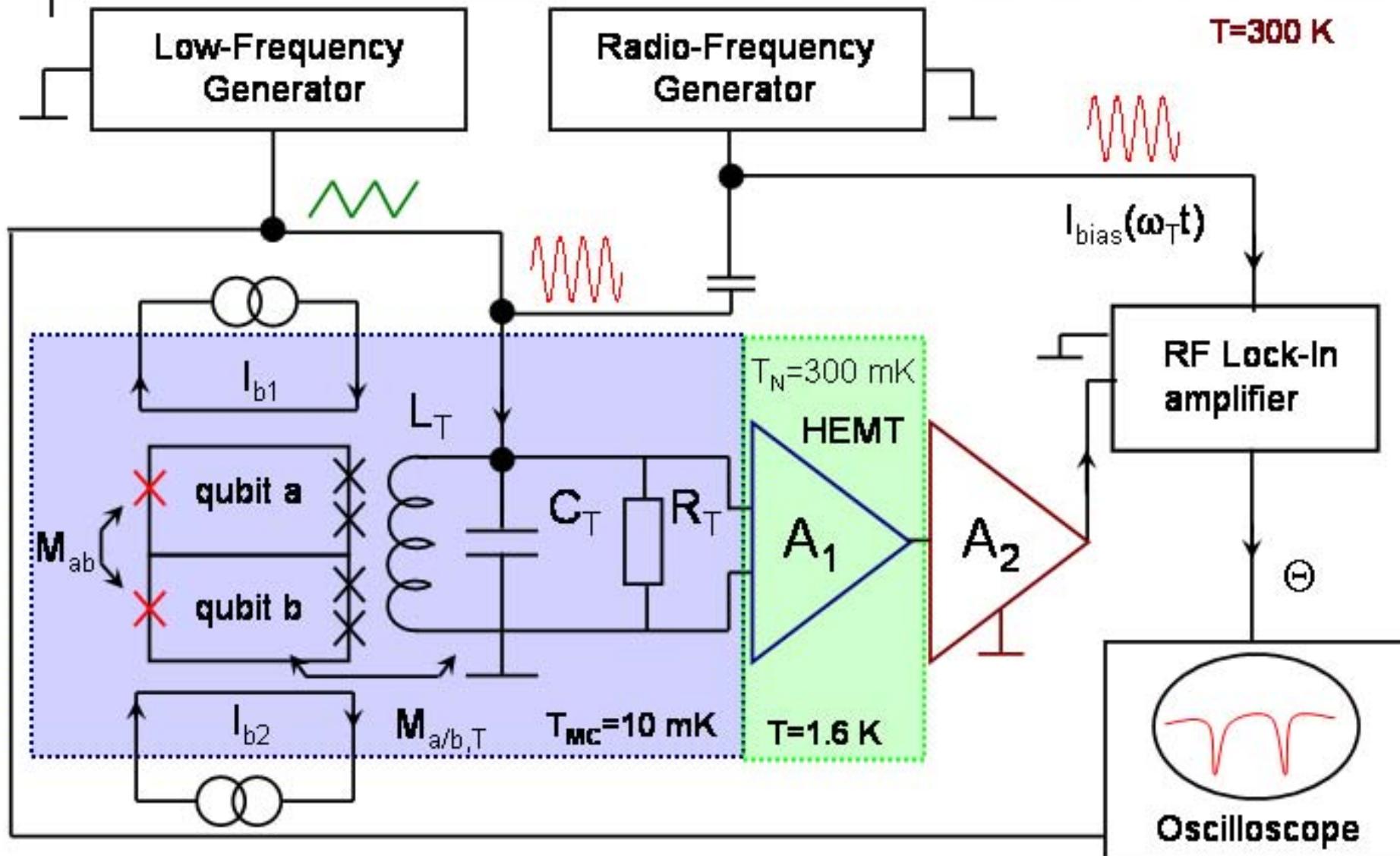


Originally proposed by L.S. Levitov et al,
[cond-mat/0108266](https://arxiv.org/abs/cond-mat/0108266)

Intuitively: The phase drop over the coupling junction leads to reduction in junctions energy of qubits and increase in Josephson energy of the junction 0.



Measurement scheme

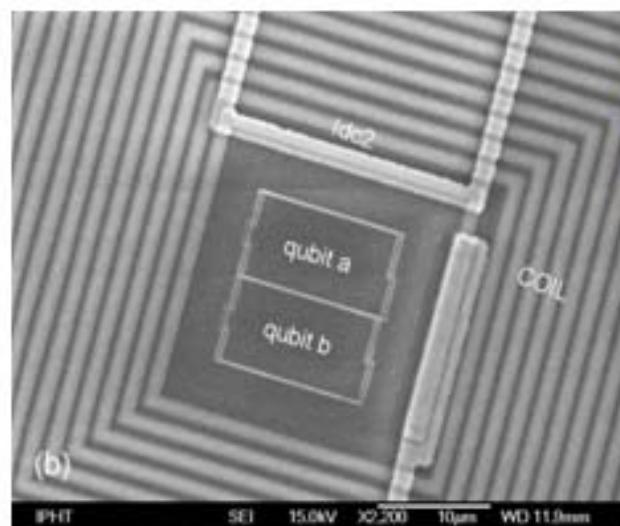
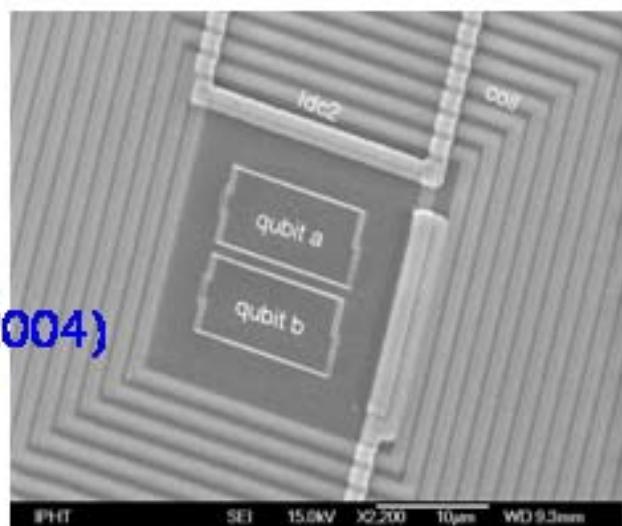


Micrographs

1.

Weak inductive
Coupling $J=20 \text{ mK}$

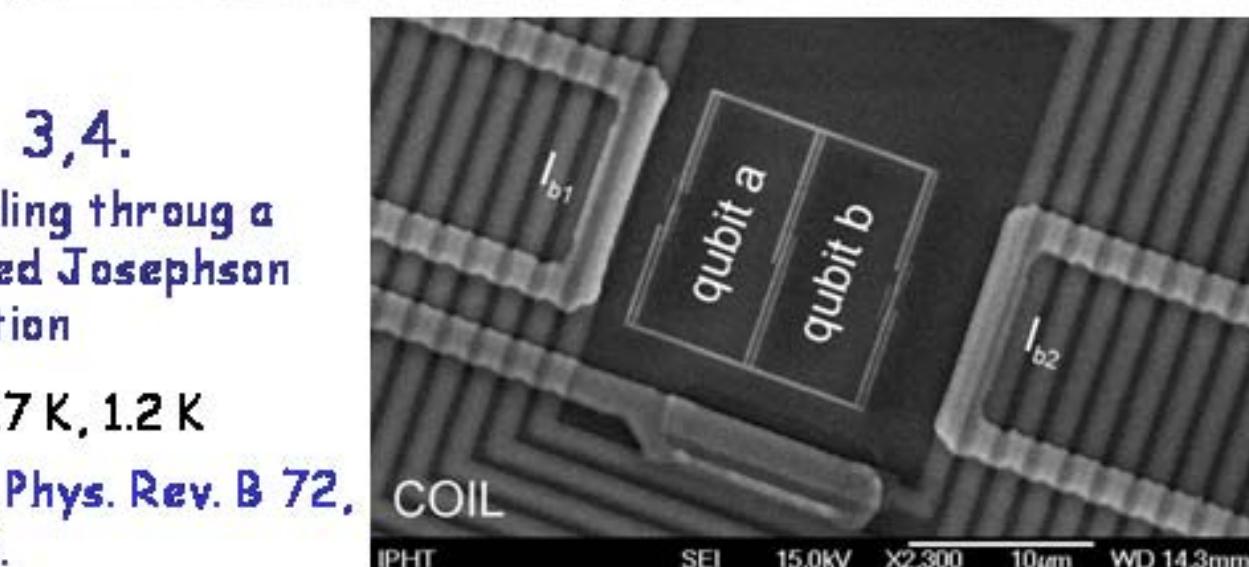
A. Izmalkov *et al.*,
PRL 93, 037003 (2004)



2.

Strong
inductive
coupling

$J=150 \text{ mK}$



3,4.

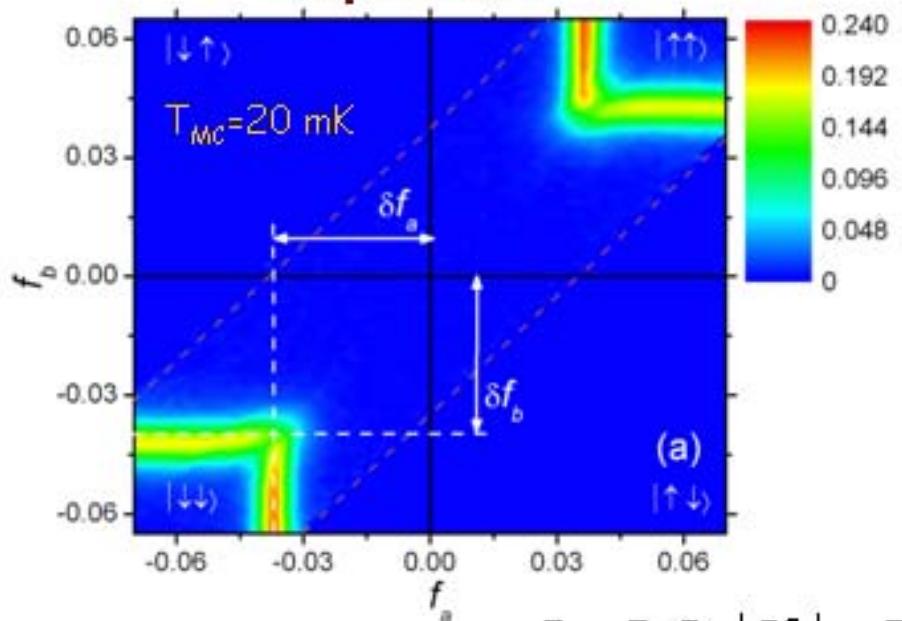
Coupling through a
shared Josephson
junction

$J=0.7 \text{ K}, 1.2 \text{ K}$

M. Grajcar, *et al.*, Phys. Rev. B 72,
R020503, (2005).

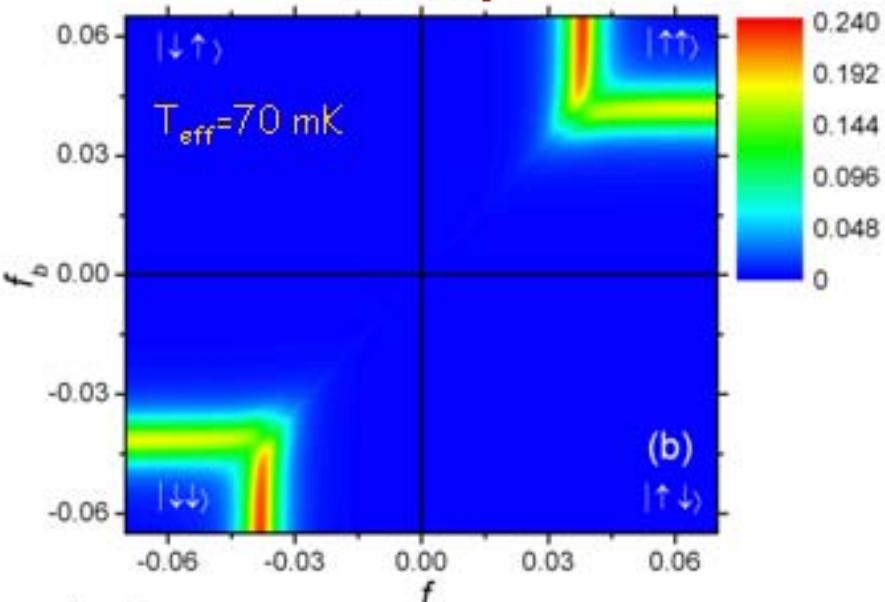
Coupling through a shared Josephson junction. IMT flux diagram.*

Experiment



(a)

Theory



(b)

$$J = I_a \Phi_0 |\delta f_a| = I_b \Phi_0 |\delta f_b| = 700 \text{ mK}$$

Sample No.	$S_{J0}, (\mu\text{m}^2)$	$\Delta_a, (\text{mK})$	$\Delta_b, (\text{mK})$	$I_a, (\text{nA})$	$I_b, (\text{nA})$	$J, (\text{K})$
1	0.3	80	90	120	110	0.7
2	0.15	30	30	150	120	1.2

*M. Grajcar, et al., Phys. Rev. B 72, 020503 (R), (2005).

Concept of Adiabatic Quantum Computation*

1. Encode your mathematical problem into the "problem Hamiltonian"

$$H_P = \sum_{i=1}^N \epsilon_i(f_i) \sigma_{z,i} + \sum_{i=1}^N \Delta_i \sigma_{x,i} + \sum_{i < j} J_{i,j} \sigma_{z,i} \sigma_{z,j}$$

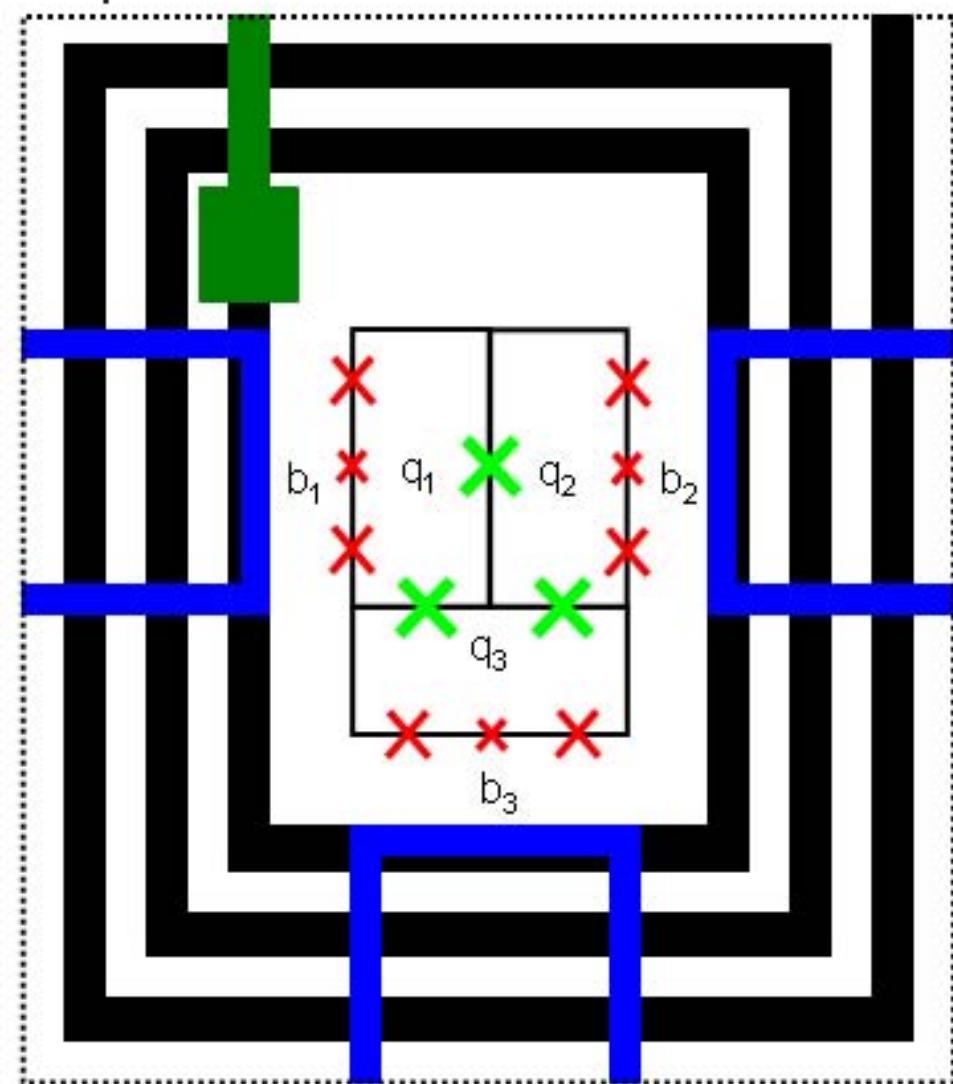
2. Make a chip with layout in accordance with the model Hamiltonian (Analogue Computer)
3. Start with initial Hamiltonian H_I in the known ground state $|I\rangle$

$$H_I = \sum_{i=1}^N \epsilon_i(f_i) \sigma_{z,i}$$

4. Adiabatic evolution from $|I\rangle$ to the unknown ground state $|g\rangle$ of H_P by changing the bias of individual qubits adiabatically
5. Readout the ground state $|g\rangle$ of H_P

* E. Farhi, J. Goldstone, S. Gutmann, M. Sipser, quant-ph/0001106.

Sample layout

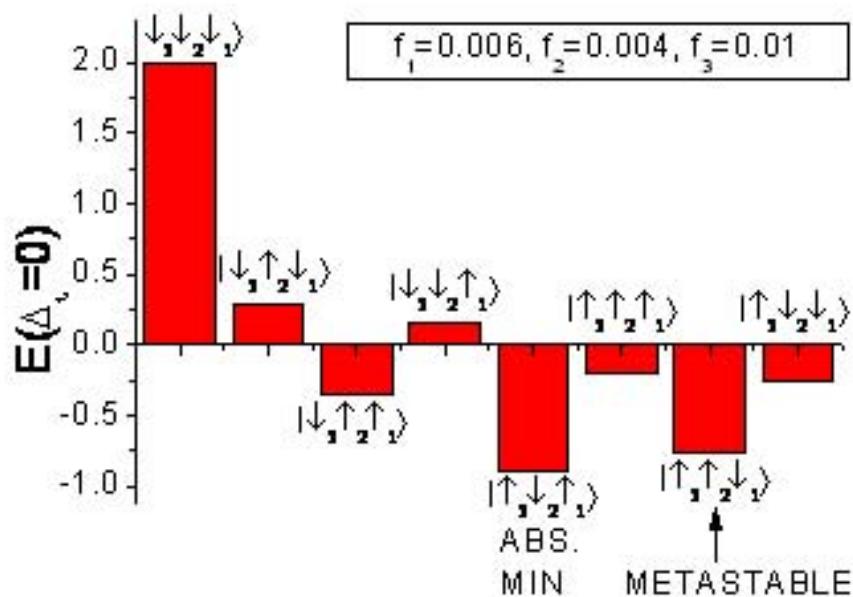


$$J_{12}=J_{13}=J_{23}=0.3 \text{ K}$$

$$\Delta_1=\Delta_2=\Delta_3=96 \text{ mK}$$

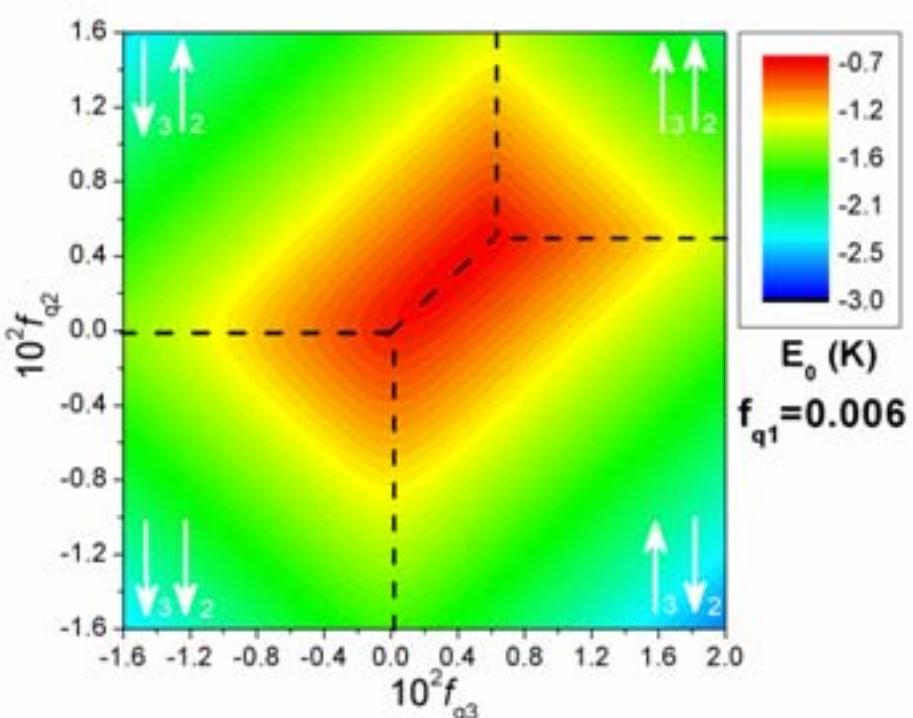
$$I_{p1}=I_{p3}=350 \text{ nA}, I_{p2}=420 \text{ nA}$$

If $f_1=6m\Phi_0, f_2=4m\Phi_0, f_3=10m\Phi_0$
 solution is $|\uparrow\downarrow\uparrow\rangle$

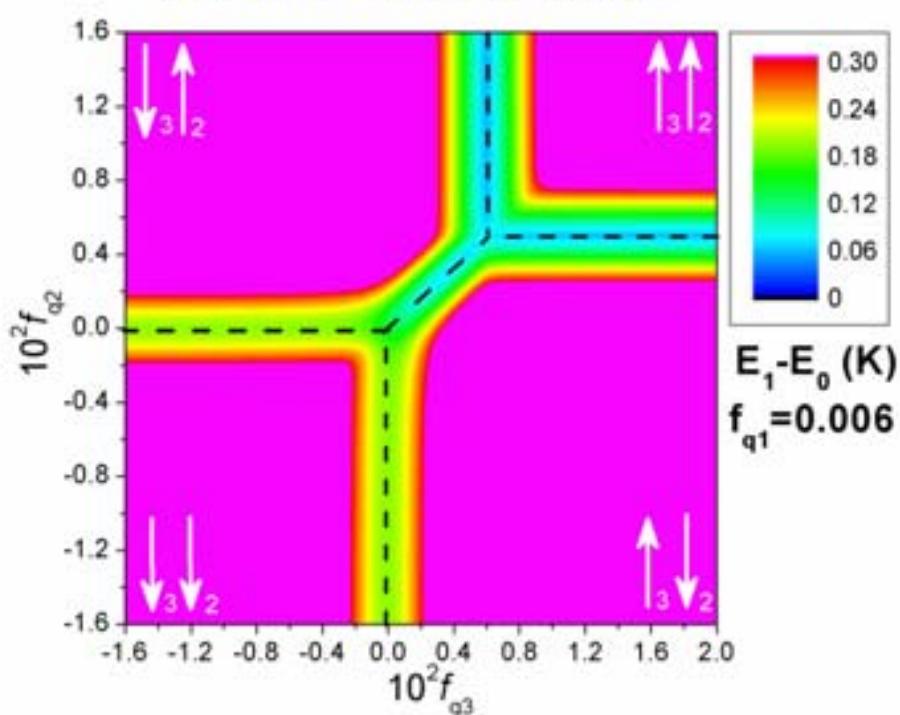


Adiabatic evolution

Energy of the ground state



Spacing between ground and first excited states

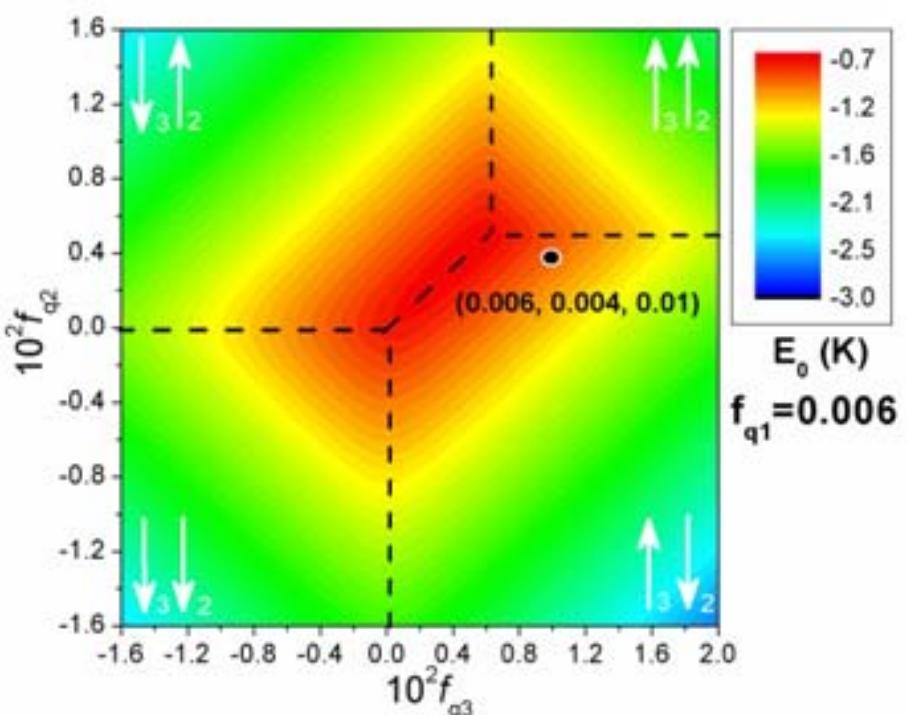


— — — Max curvature of the ground state

- boundary of the classical stability diagram

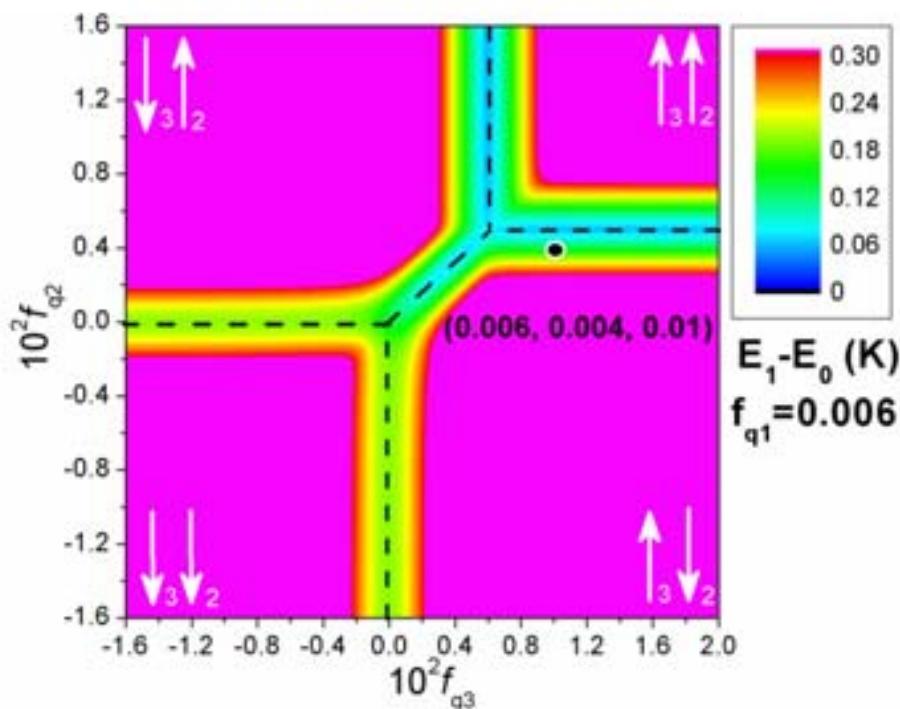
Adiabatic evolution

Energy of the ground state



— — — Max curvature of the ground state

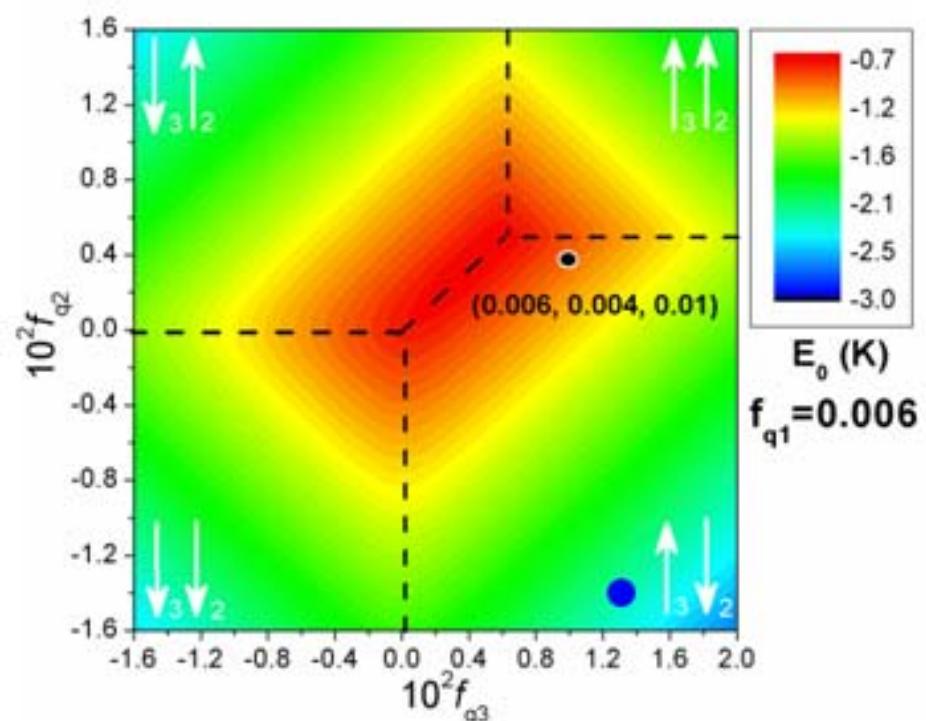
Spacing between ground and first excited states



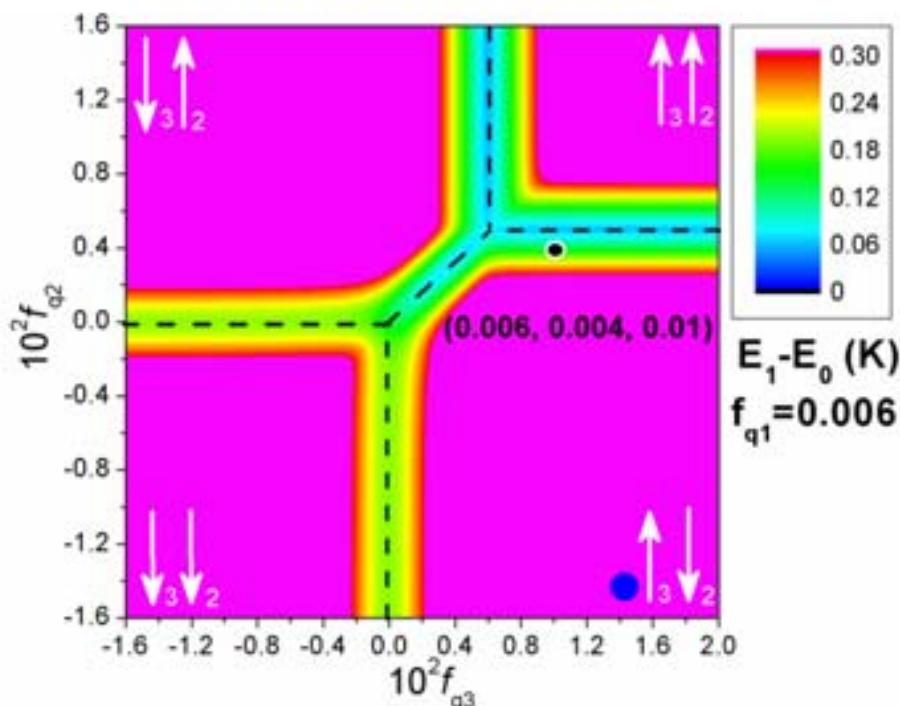
If $f_1 = 6m\Phi_0$, $f_2 = 4mF_0$, $f_3 = 10mF_0$
 solution is $|\uparrow\downarrow\uparrow\rangle$

Adiabatic evolution

Energy of the ground state



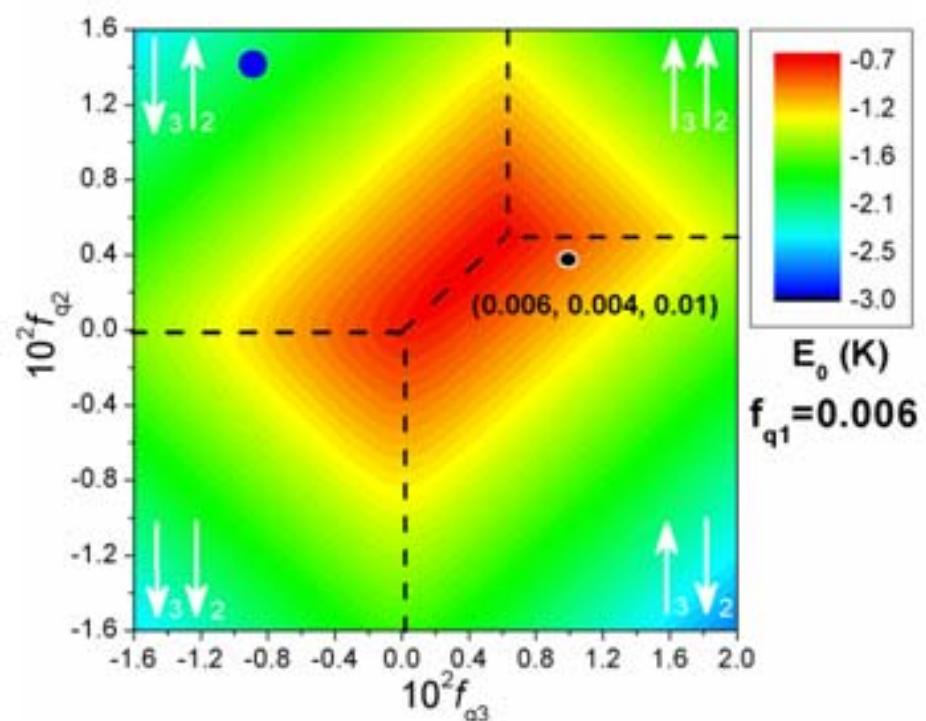
Spacing between ground and first excited states



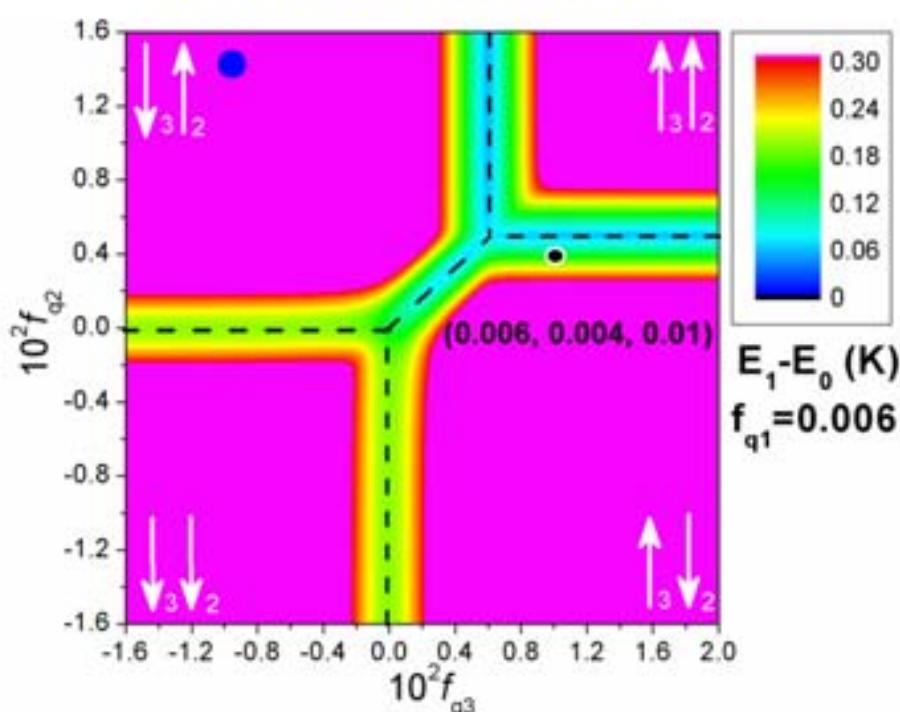
— — — Max curvature of the ground state

Adiabatic evolution

Energy of the ground state



Spacing between ground and first excited states

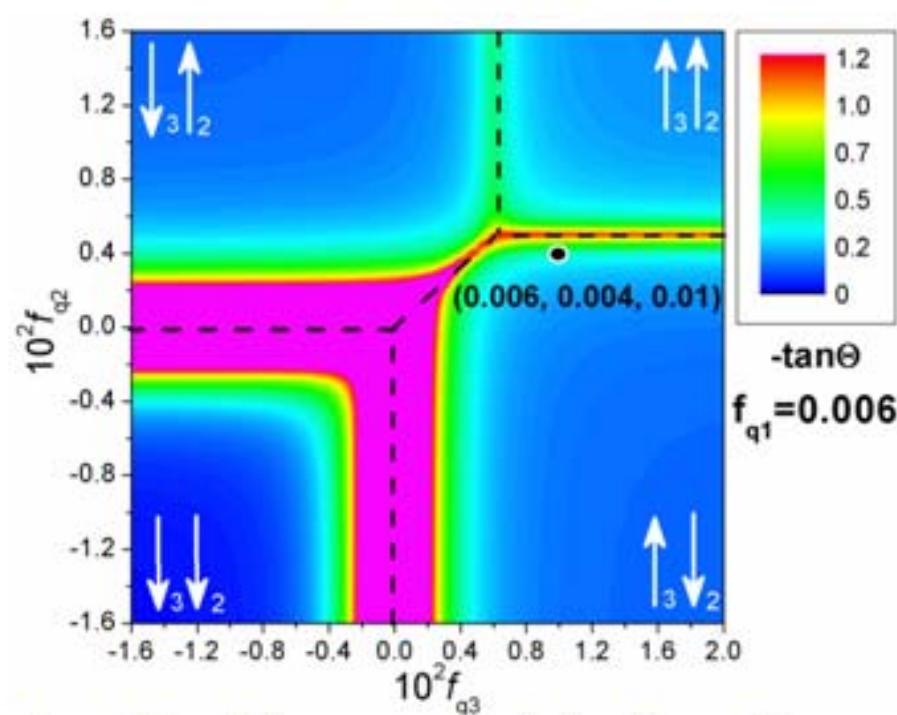
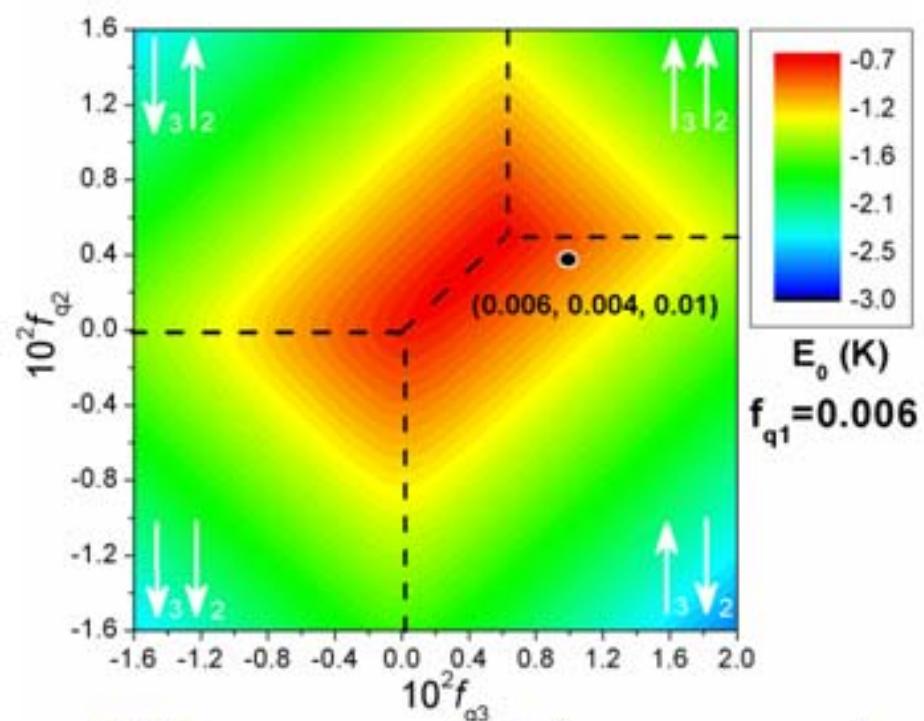


— — — Max curvature of the ground state

Readout (Flux map)

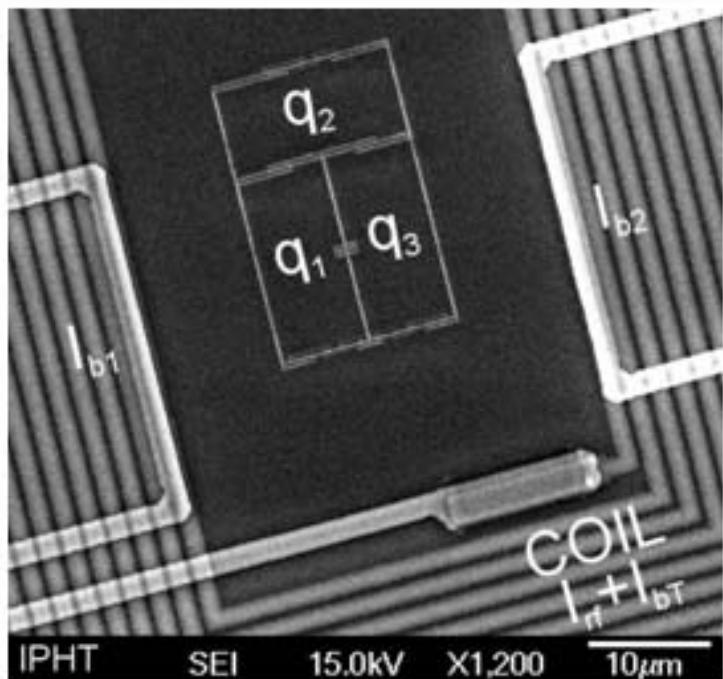
M. Grajcar, A. Izmalkov, E. Il'ichev, *Phys. Rev. B* **71**, 144501 (2005)

T=10 mK



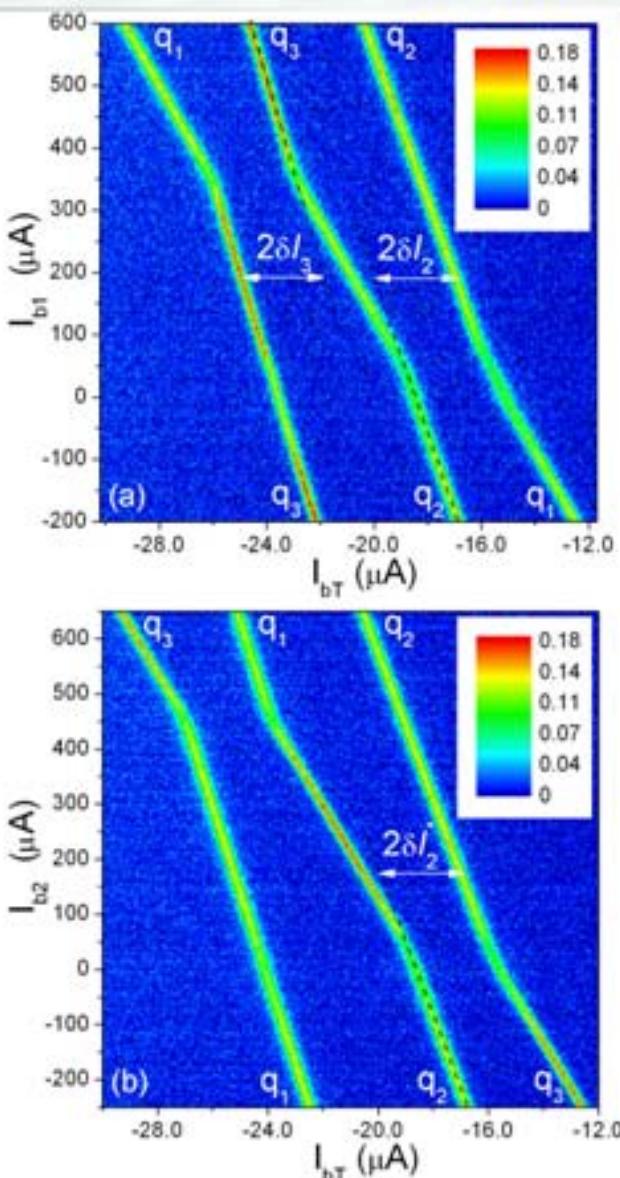
- IMT response depicts max curvature of multiqubit ground state, therefore giving solution AC
- Flux map readout should be quite fast and T_{eff} should be small, in order to prevent temperature transitions to first excited state

Three qubits, preliminary experimental data



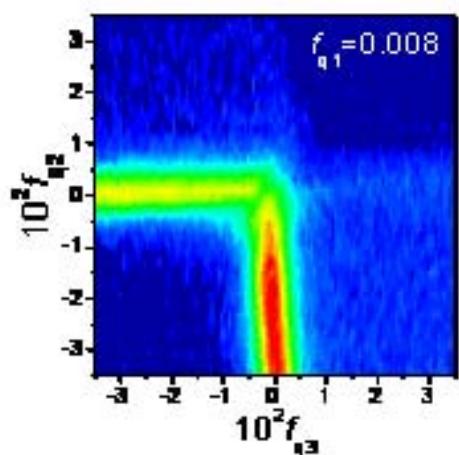
Sample parameters:

$J_{12}=J_{13}=J_{23}=0.61 \text{ K}$,
 $\Delta_1=\Delta_2=\Delta_3=70 \text{ mK}$,
 $I_{p1}=I_{p2}=115 \text{ nA}$, $I_{p3}=125 \text{ nA}$

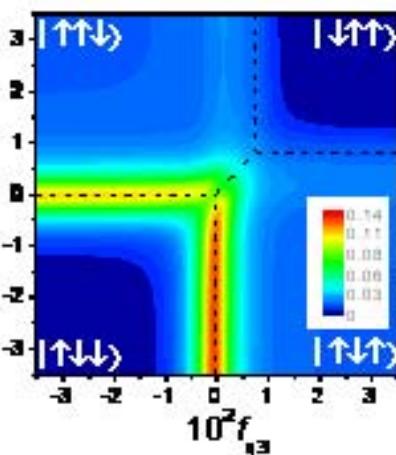


3D susceptibility of three qubit system

Experiment

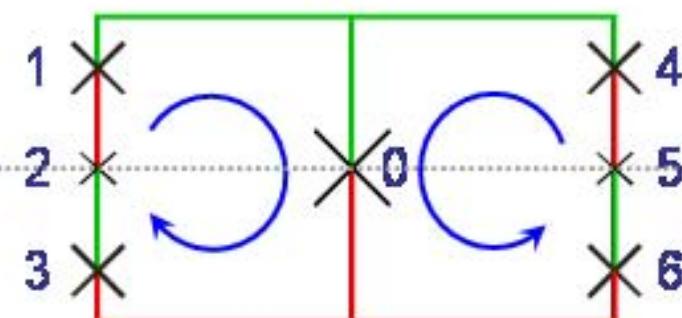


Theory

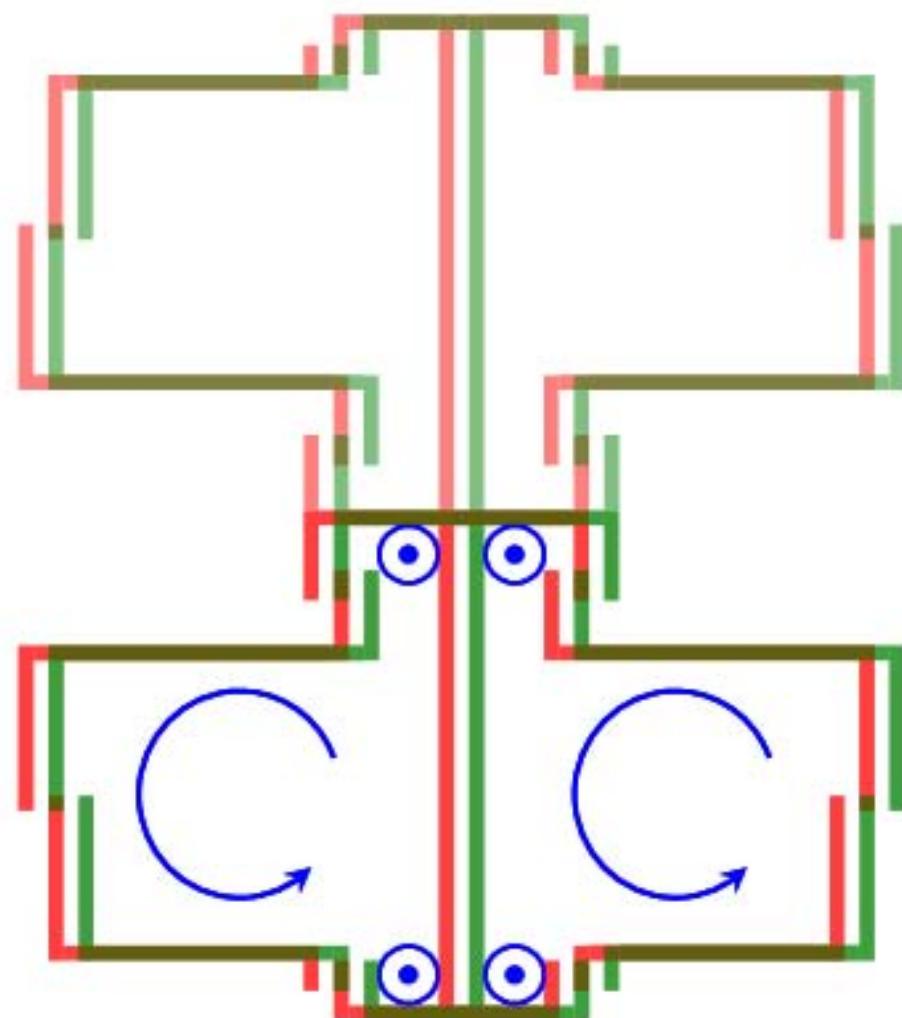
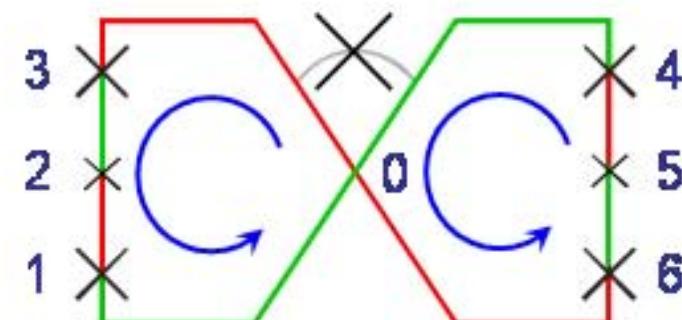


Four qubit sample with mixed couplings. Layout

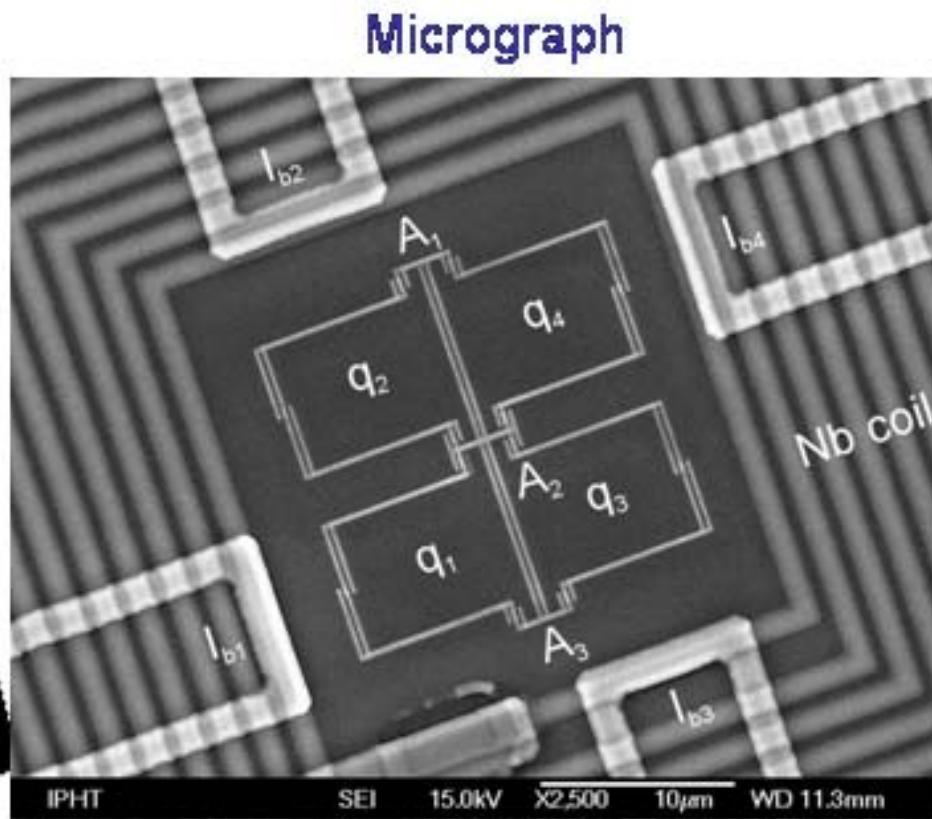
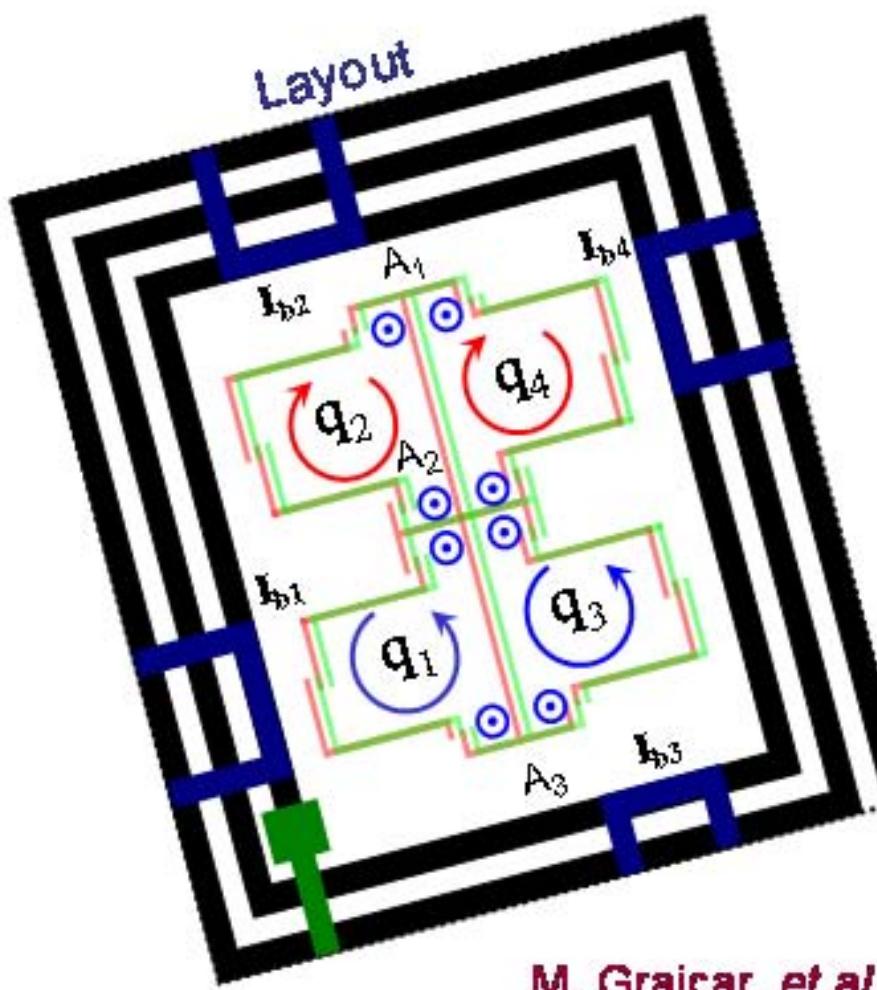
AFM



FM

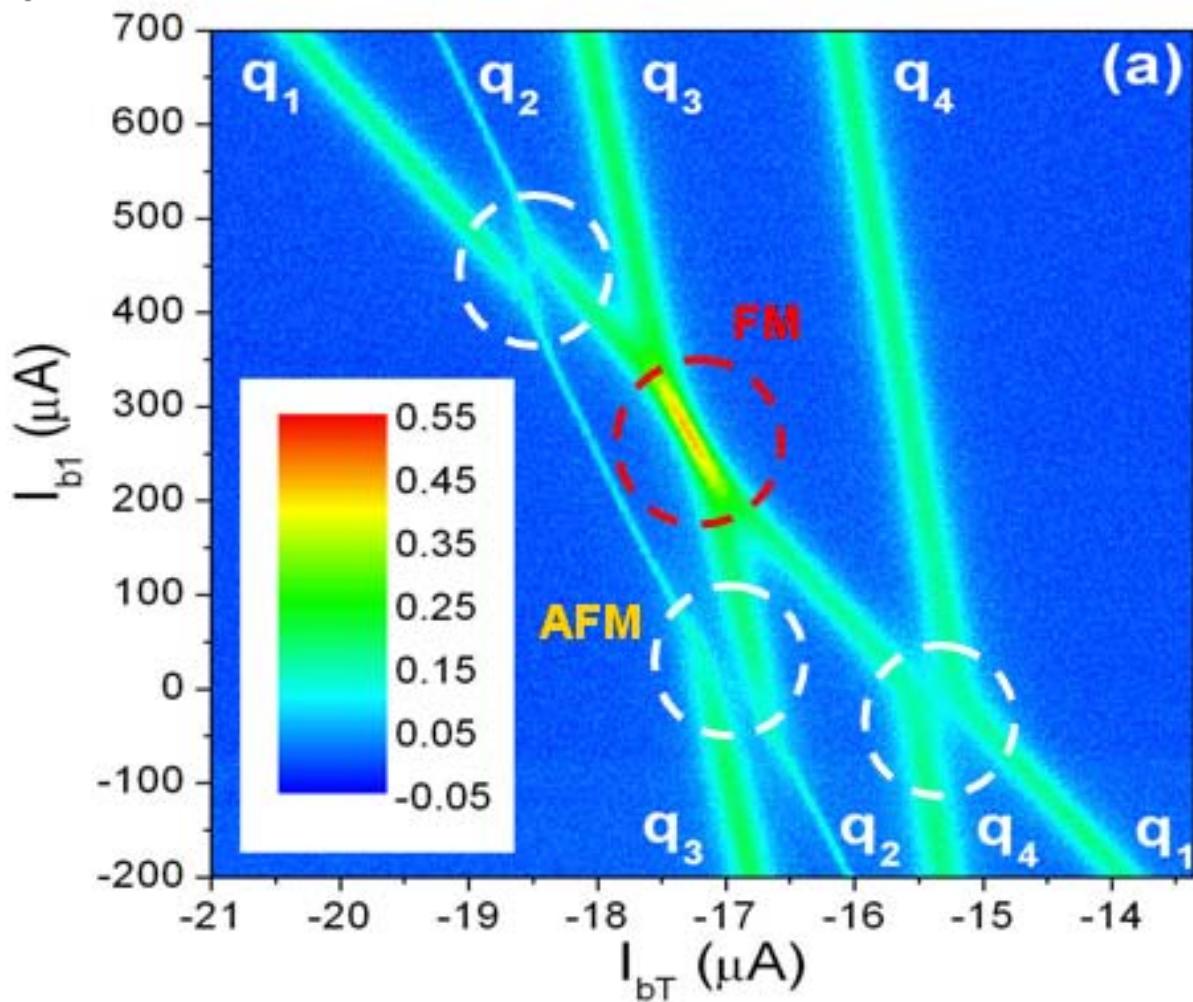


Four qubit sample. Micrograph



M. Grajcar, et al., cond-mat/0509557.

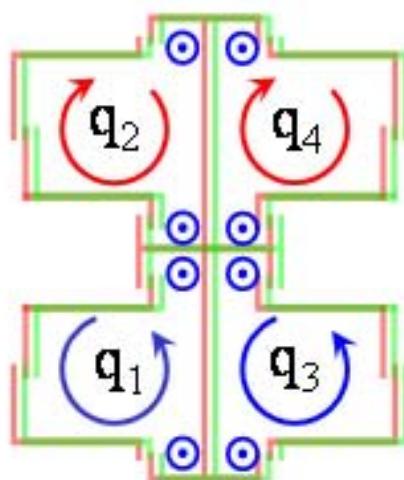
Anti-Ferromagnetic (AFM) and Ferromagnetic (FM) Coupling



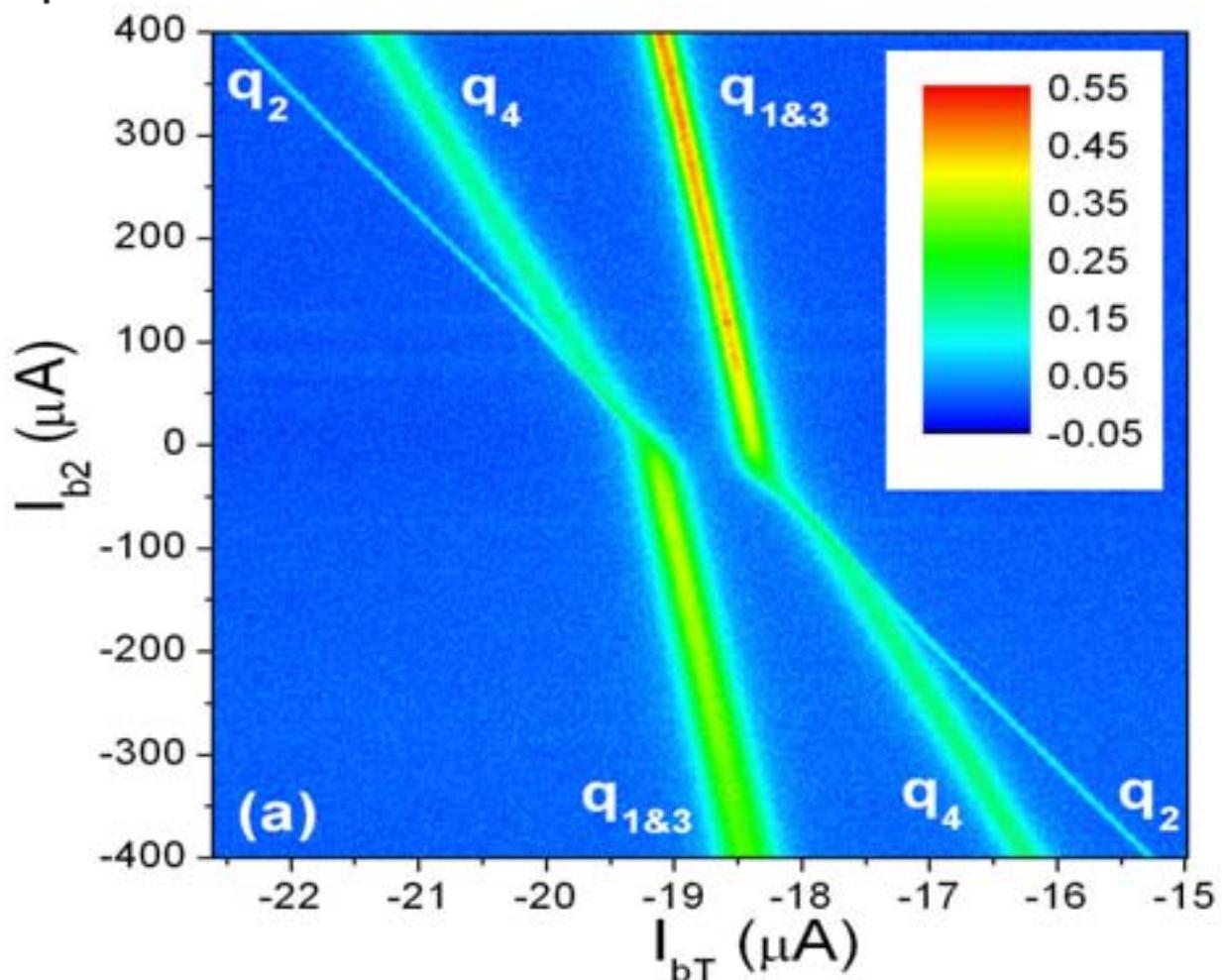
$$I_{b2} = 10 \mu\text{A}$$

$$I_{b3} = 0 \mu\text{A}$$

$$I_{b4} = 250 \mu\text{A}$$



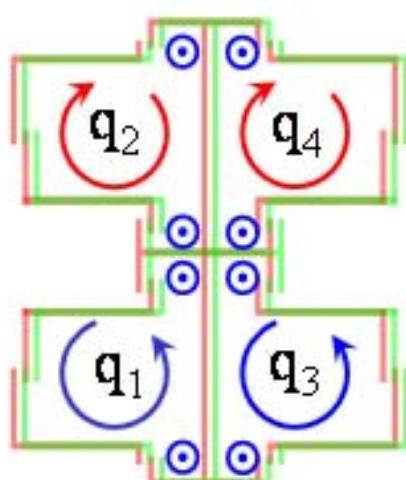
AFM interaction between FM interacting qubit pairs



$$I_{b1} = -400 \mu\text{A}$$

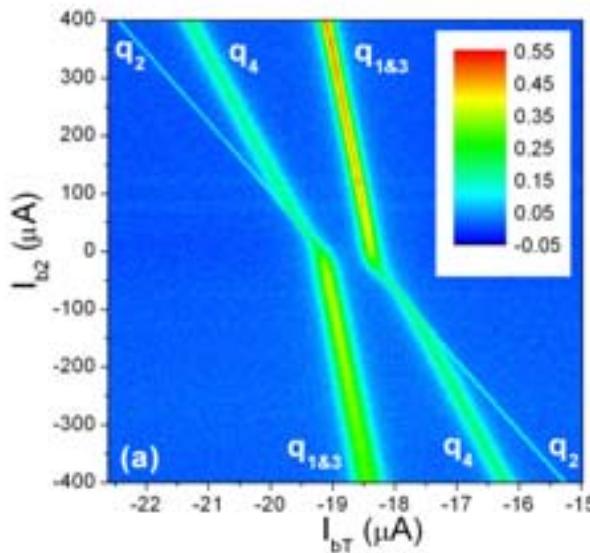
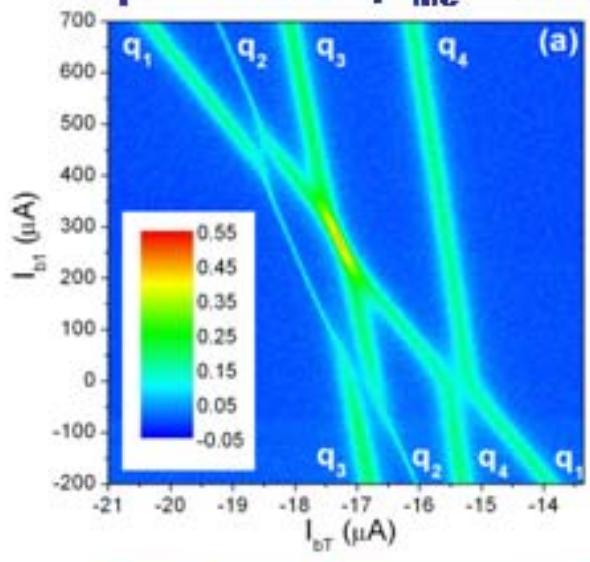
$$I_{b3} = 0 \mu\text{A}$$

$$I_{b4} = -50 \mu\text{A}$$

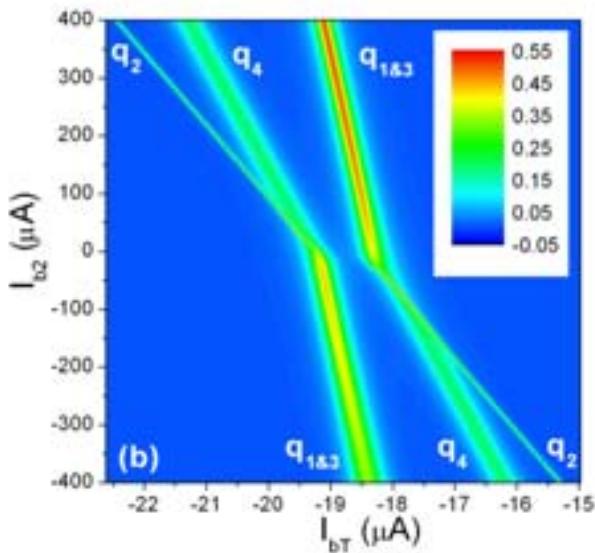
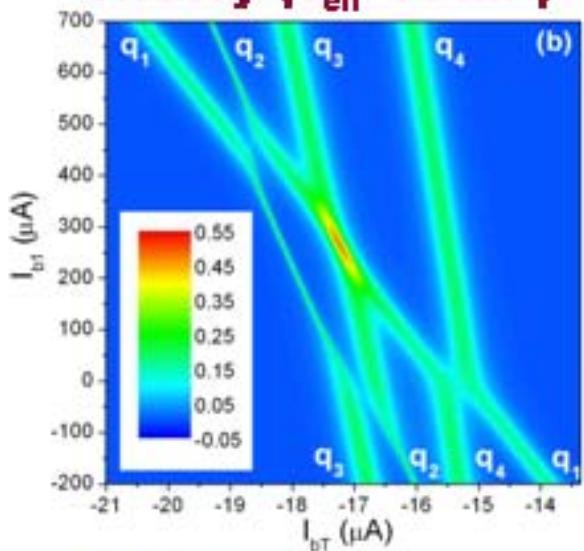


Theoretical fits

Experiment ($T_{MC}=10$ mK)

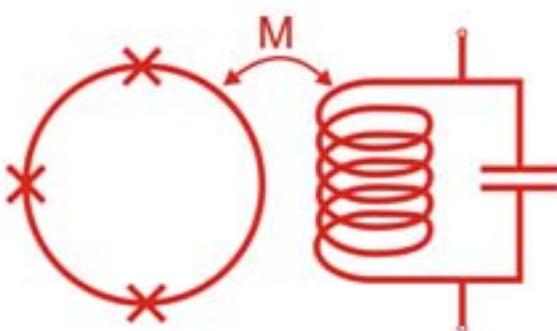


Theory ($T_{eff}=70$ mK)



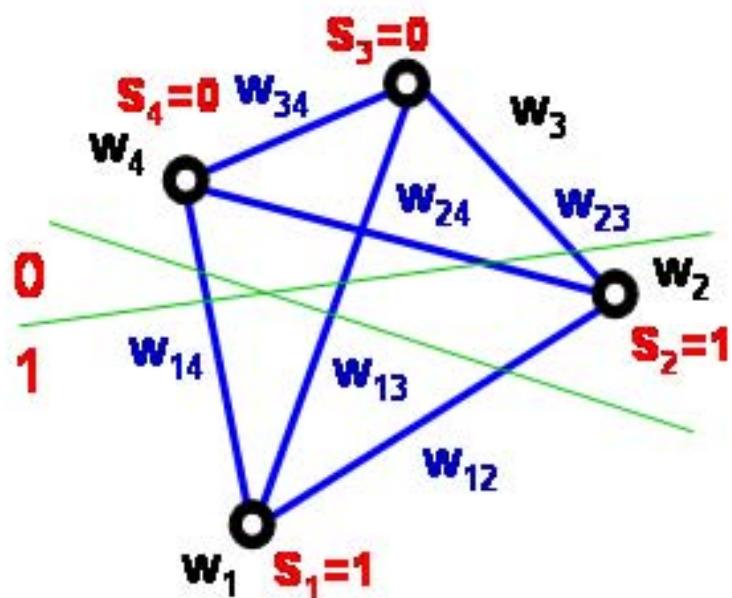
Measurements by making use of:

- (PTB charge qubit) - LC circuit
- (Chalmers charge qubit) - RF SET
- (Sakke quantronium) - LC circuit with JJ
- (Delft flux qubit) - LC circuit with SQUID
- (Yale charge qubit) - strip resonator
- Parametric transduser



1. Encoding MAXCUT problem

Example for 4 nodes



$$P(\langle s \rangle) = \sum_i w_i s_i + \sum_{i,j} s_i (1 - s_j) w_{i,j}$$

$$\max(P) \text{ for } |s\rangle_{solution} = |0011\rangle \equiv |\downarrow_4 \downarrow_3 \uparrow_2 \uparrow_1\rangle$$

$$H_P = \sum_{i=1}^N \varepsilon_i(f_i) \sigma_{z,i} + \sum_{i < j}^N J_{i,j} \sigma_{z,i} \sigma_{z,j} + \sum_{i=1}^N \Delta_i \sigma_{x,i}$$

Payoff function is encoded in Hamiltonian H_P if $\Delta_i \ll J_{i,j}$ and

$$\varepsilon_i \equiv I_{pi} \Phi_0 f_i = -w_i / 2; \quad J_{i,j} = w_{i,j} / 2$$

$$f_i = \Phi_i / \Phi_0 - 0.5 \quad \text{- flux through qubit } i$$

H_P – The MAXCUT problem Hamiltonian

$$H_P |g\rangle = E_g |g\rangle$$